Dhirubhai Ambani Institute of Information and Communication Technology

Ph.D.

Mathematics

Sample Question Paper

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1. For a Hermitian matrix of order 3, all its eigenvalues are ...

   (a) purely imaginary numbers

   (b) real numbers

   (c) positive numbers

   (d) negative numbers

   (e) of absolute value 1

2. How many 2×2 invertible matrices are there with entries from $\mathbb{Z}_2$?

   (a) 2

   (b) 4

   (c) 6

   (d) 8

   (e) 15

3. The number of distinct 1-dimensional subspaces of $V = \mathbb{Z}_2^3$ is ...

   (a) 2

   (b) 3

   (c) 5

   (d) 6

   (e) 7

4. Suppose A, B are $n \times n$ matrices. Then A and B must be similar if ....

   (a) they have same characteristic polynomial.

   (b) they have same minimal polynomial.

   (c) they have same eigenvalues.

   (d) they are diagonalizable.

   (e) none of these.

5. The number of non-isomorphic groups of order 4 is

   (a) 1

   (b) 2

(c) 3

(d) 4

(e) 6

6. The number of generators of a cyclic group of order 12 is

   (a) 1

   (b) 2

   (c) 3

   (d) 4

   (e) 12

7. Let G be a group of order 30. Then which of the following is always true?

   (a) G contains elements of order $2, 3, 5$.

   (b) G may not contain any element of order 3.

   (c) G may not contain any element of order 2.

   (d) G may not contain any element of order 5.

   (e) None of these.

8. In the ring $\mathbb{Z}[x]$, the ideal generated by x is ....

   (a) prime ideal but not maximal ideal.

   (b) maximal ideal but not prime ideal.

   (c) both prime ideal and maximal ideal.

   (d) neither prime ideal nor maximal ideal.

   (e) not a principal ideal.

9. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ is ....

   (a) convergent for $p = 1$ but not absolutely convergent.

   (b) absolutely convergent for all $p \in \mathbb{R}$.

   (c) absolutely divergent for all $p \in \mathbb{R}$.

   (d) divergent for all $p \in \mathbb{R}$.

   (e) divergent only for $p = 1$.

10. Let $f_n : \mathbb{R} \to \mathbb{R}$ be defined as $f_n(x) = \frac{\sin nx}{\sqrt{n}}$ and $g_n : [0, 1] \to \mathbb{R}$ be defined as $g_n(x) = x^n$. The which of the following is true?

    (a) Both $\{f_n(x)\}$ and $\{g_n(x)\}$ are uniformly convergent.

    (b) $\{f_n(x)\}$ is uniformly convergent but $\{g_n(x)\}$ is not uniformly convergent.

(c) $\{g_n(x)\}$ is uniformly convergent but $\{f_n(x)\}$ is not uniformly convergent.

(d) None of $\{f_n(x)\}$ and $\{g_n(x)\}$ is pointwise convergent.

(e) Both $\{f_n(x)\}$ and $\{g_n(x)\}$ are pointwise convergent but not uniformly convergent.

11. Let (X, d) be an arbitrary metric space. Then which of the following is NOT true?

(a) Each compact subset of X is closed and bounded.

(b) If S is an infinite compact subset of X, then S has a limit point.

(c) If $X = \mathbb{R}^n$, then every closed bounded subset of X is compact.

(d) Each closed bounded subset of X is compact.

(e) If X is compact, then each closed subset of X is compact.

12. Suppose $f : \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = \lfloor x + a \rfloor$, where $a \in \mathbb{R}$ and $\lfloor x \rfloor$ is the greatest integer less than equal to x. Then which of the following statement is true?

(a) The function $f(x)$ is continuous at $x = -a$ but not differentiable at $x = -a$.

(b) The function $f(x)$ is not continuous at $x = -a$ but differentiable at $x = -a$.

(c) The function $f(x)$ is continuous and differentiable at $x = -a$.

(d) The function $f(x)$ is neither continuous nor differentiable at $x = -a$.

(e) None of these.

13. Suppose X represents the outcome when a fair die is rolled. What is variance of $2X + 100$?

(a) $\frac{35}{3}$

(b) 107

(c) $\frac{335}{3}$

(d) $\frac{635}{6}$

(e) None of the above

14. Suppose X is a random variable uniformly distributed over $(0, 1)$. Then what is expectation of X^2?

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{12}$

(d) $\frac{1}{4}$

(e) None of the above

15. Suppose X, Y are random variables. $Cov(X, Y), E(X)$ denote the covariance of X, Y and expectation of X respectively. Then which of the following is true?

(a) $Cov(2X, Y) = 4Cov(X, Y)$.

(b) $E(XY) = 0$ if X, Y are independent.

(c) $Cov(X, Y) = 0$ if X, Y are independent.

(d) $Cov(X, X) = E(X^2)$.

(e) None of these.

16. A coin is flipped twice. Suppose all the points in the sample space are equally likely. Then what is the conditional probability that both flips are tails given that the first flip is a tail?

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{3}{4}$

(d) 1

(e) None of these

17. The radius of convergence of $f(z) = \sum_{n=0}^{\infty} z^{2^n}$ is

(a) 1

(b) 2

(c) 4

(d) infinity

(e) 0

18. Let w be a singularity of $f(z) = \log z$. Then w is a ...

(a) pole of order more than 1.

(b) branch point.

(c) essential singularity.

(d) simple pole.

(e) none of these.

19. Which of the following is true?

(a) $\sin(z)$ is an entire and bounded function.

(b) $\sin(z)$ is neither entire nor bounded function.

(c) $\sin(z)$ is an entire and unbounded function.

(d) $\sin(z)$ is not entire but bounded function.

(e) none of these.

20. Consider $C = \{z \in \mathbb{C} : |z| = 2\}$. Then the value of $\int_C \frac{1}{z^2+1}$ is ...

(a) $2\pi i$

(b) πi

(c) π

(d) $-\pi$

(e) 0