Dhirubhai Ambani Institute of Information and Communication Technology

Ph.D.

Physics

Sample Question Paper

Physical constants and units

$G = 6.67 \times 10^{-11}\text{m}^3/\text{kg s}^2$	$g = 9.8\text{m/s}^2$
$\epsilon_0 = 8.85 \times 10^{-12}\text{C}^2/\text{Nm}^2$	$\mu_0 = 4\pi \times 10^{-7}\text{N/A}^2$
electron charge $e = 1.6 \times 10^{-19}$ C	Speed of light $c = 3 \times 10^8$ m/sec
Planck's constant $h = 6.626 \times 10^{-34}$ joule-sec	Boltzmann's constant $k = 1.38 \times 10^{-23}$ joule/K
Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W/ m^2K^4	Wien's constant $= 2.9 \times 10^{-3}$m K
Gas constant $R = 8.31441$ joule/mole K	Avogadro number $N = 6.023 \times 10^{23}$
Proton mass $m_p = 1.673 \times 10^{-27}kg = 1.007277$u	Neutron mass $m_n = 1.675 \times 10^{-27}kg = 1.008665$u
Electron mass $m_e = 9.109 \times 10^{-31}kg = 0.00055$u	
1 Gauss = .0001 Tesla	$1\text{eV} = 1.6 \times 10^{-19}$ joule
1 u (amu) =931 MeV	$1\text{A}^\circ = 10^{-10}$m

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1. The value of the integral $\int_0^a \int_0^a \delta(x-y)dxdy$ is

   (A) 1

   (B) a^2

   (C) a

   (D) ∞

   (E) None of the above

2. Consider the following partial differential equation

$$\frac{\partial^2\Phi(x,y)}{\partial x^2} + \frac{\partial^2\Phi(x,y)}{\partial y^2} = 0$$

   Which of the following is a solution to this equation:

   (A) $\sin k(x+y)$

   (B) $\sin(k_1x + k_2y) \quad k_1 \neq k_2$

   (C) $e^{kx} \sin ky$

   (D) $e^{k(x+y)}$

   (E) None of the above

3. A box contains 1200 tokens numbered 1 to 1200. A token is drawn at random. It is found that the token number is a multiple of 3. Given this information what is the probability that the token number is also a multiple of 5.

   (A) 1/6

   (B) 1/5

   (C) 1/15

   (D) 1/3

   (E) None of the above

4. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$, $T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix}$, $T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix}$ then

   (A) dim (kernel T) $= 2$
   (B) rank $T = 0$
   (C) a basis of kernel T is $\left\{\begin{pmatrix}-1\\1\\1\end{pmatrix}\right\}$
   (D) range $T = \mathbb{R}^3$
   (E) None of the above

5. If $z^8 = 1$ and in the set $\{1, z, z^2, z^3, z^4, z^5, z^6, z^7\}$ all the elements are distinct, then

   (A) $1, z^2, z^4, z^6$ are real
   (B) all the elements are real.
   (C) z, z^3, z^5 and z^7 are pure imaginary.
   (D) Only 1 and z^4 are real.
   (E) None of the above

6. The effective potential of a particle in a central force field is described by $V(r) = -\frac{5}{r} + \frac{4}{r^2}$. If the energy of a particle in this field is $E = -1$ then the nearest distance of the particle from the center of force is

   (A) 3
   (B) 2
   (C) 4
   (D) 1
   (E) None of the above

7. A particle moves in a central force field with the center of force at the origin. If the velocity of the particle at $\vec{r}_0$ is $\vec{v}$ then the trajectory of the particle is contained in the plane described by

   (A) $(\vec{r} \times \vec{r}_0) \cdot \vec{v} = 0$
   (B) $(\vec{r} \times \vec{v}) \cdot \vec{r}_0 = 0$
   (C) $(\vec{r}_0 \times \vec{v}) \cdot \vec{r} = 0$
   (D) $(\vec{r} - \vec{r}_0) \cdot \vec{v} = 0$
   (E) None of the above

8. The total momentum of a system of particles

   (A) is equal to the momentum of the center of mass of the system.
   (B) is equal to the total momentum of the particles in the center of mass of the system.

(C) is equal to the sum of the momentum of the center of mass of the system and the total momentum of the particles in the center of mass of the system.

(D) can change if the particles in the system exert force on each other.

(E) None of the above.

9. The Lagrangian for a particle of mass m is given as $L = \frac{1}{2}m\dot{x}^2 - mg(h - x\sin\theta)$ where g and θ are constants. The equation of motion is

   (A) $\ddot{x}\sin\theta = g$

   (B) $\ddot{x} = g$

   (C) $\dot{x}\sin\theta = gt$

   (D) $\ddot{x} = g\sin\theta$

   (E) None of the above

10. A particle is trapped in a box of length L. Then the expectation of its momentum in first excited state is:

   (A) π/L

   (B) 0

   (C) Infinity

   (D) $\sqrt{\frac{\pi}{L}}$

   (E) None of the above

11. Suppose the position uncertainty of an electron is zero. Then it's momentum uncertainty is:

   (A) 0

   (B) Greater than $\hbar/2$

   (C) Less tha $\hbar/2$

   (D) Infinity

   (E) None of the above

12. Consider two states $|\psi\rangle = 2i|\phi_1\rangle + a|\phi_2\rangle - |\phi_3\rangle$ and $|\chi\rangle = 5|\phi_1\rangle + 6|\phi_2\rangle + 4|\phi_3\rangle$ where $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$ are orthonormal to each other and a is a constant. Then $|\psi\rangle$ and $|\chi\rangle$ will be orthogonal if a is equal to:

   (A) $\frac{2-5i}{3}$

   (B) $\frac{1-5i}{3}$

   (C) $\frac{2-5i}{4}$

   (D) $\frac{3-5i}{2}$

   (E) None of the above.

13. Consider the position operator x and the momentum operator p_x. Then using the result $[x, p_x] = i\hbar$, show that $[x^2, p_x] =$?:

   (A) $\frac{i\hbar}{2}$

(B) $\sqrt{2}i\hbar x$

(C) $2i\hbar x$

(D) $-i\hbar x$

(E) None of the above

14. Consider an atom as containing a point charge $+Q$ at its center surrounded by a uniform volume distribution of negative charge $-Q$ within a sphere of radius R. The magnitude of the electric field at a distance r from the center at a point inside the atom ($r < R$) is

(A) $\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$

(B) $\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r^2}{R^4}\right)$

(C) $\frac{Q}{4\pi\epsilon_0}\left(\frac{r}{R^3} - \frac{1}{r^2}\right)$

(D) $\frac{Q}{4\pi\epsilon_0}\left(\frac{r^2}{R^4} - \frac{1}{r^2}\right)$

(E) None of the above

15. An infinite conducting sheet has a surface charge density of 2 μCm^{-2} on one side. How far apart are the equipotential surfaces whose potentials differ by 20 V?

(A) 177.1 μm

(B) 177.1mm

(C) 88.5 μm

(D) 88.5 mm

(E) None of the above

16. In a conducting medium,

$$\vec{H} = \frac{\vec{B}}{\mu_0} = y^2 z\hat{x} + 2(x+1)yz\hat{y} - (x+1)z^2\hat{z} \text{ Am}^{-1}.$$

The current density $\vec{J}$ at $(1, 0, -3)$ is

(A) $9\hat{y}$ Am^{-2}

(B) $-4\hat{x} + 9\hat{y}$ Am^{-2}

(C) $9\hat{z}$ Am^{-2}

(D) $-9\hat{y}$ Am^{-2}

(E) None of the above

17. Given $\vec{E} = 30\pi \exp\left(10^8 t + kz\right)\hat{x}$ Vm^{-1} and $\vec{B} = B_0 \exp\left(10^8 t + kz\right)\hat{y}\ \frac{\text{N}}{\text{Am}}$ in free space, $H_0 = B_0/\mu_0$ is given by

(A) $-\frac{1}{4}$ Am^{-1}

(B) $\frac{1}{4}$ Am^{-1}

(C) $-\frac{1}{2}$ Am^{-1}

(D) $\frac{1}{2}$ Am^{-1}

(E) None of the above

18. Two containers of the same volume V contain ideal gasses A and B at the same pressure P and temperature T. The gasses are mixed and put in a container of volume V. Then which of the following is true about the pressure and temperature of the mixed gas ?

    (A) $2P$ and T
    (B) $2P$ and $2T$
    (C) P and $2T$
    (D) P and T
    (E) None of the above

19. The molar specific heat of an ideal gas under constant volume and constant pressure is C_V and C_P. When one mole of an ideal gas in a cylinder of volume V is heated from temperature T_1 to T_2 the heat supplied to the gas is

    (A) $C_V(T_2 - T_1)$
    (B) $C_P(T_2 - T_1)$
    (C) $(C_V + R)(T_2 - T_1)$ where R is the gas constant
    (D) $(C_P - C_V + R)(T_2 - T_1)$ where R is the gas constant
    (E) None of the above

20. The Maxwell-Boltzmann distribution of speed v of molecules of mass m at temperature T of a gas is given as $f(v) = A\left(\frac{m}{kT}\right)^{3/2} v^2 \exp\left(-\frac{1}{2}\frac{mv^2}{kT}\right)$, where A is a normalisation constant. The most probable value of v^2 is

    (A) $\frac{2kT}{m}$
    (B) $\frac{3kT}{2m}$
    (C) $\frac{5kT}{2m}$
    (D) $\frac{kT}{m}$
    (E) None of the above